REVIEW - UNIT 4

FORMULAS TO REMEMBER:

1) Vertex of the parabola $f(x) = ax^2 + bx + c$ is (h, k), where $h = \dfrac{-b}{2a}$, $k = f(h)$.

2) Properties of Exponents $(a > 0,\ a \neq 1, b > 0, b \neq 1)$:

$a^x \cdot a^y = a^{x+y}$ $\qquad \dfrac{a^x}{a^y} = a^{x-y}$ $\qquad \left(a^x\right)^y = a^{x \cdot y}$ $\qquad a^{-x} = \dfrac{1}{a^x}$ $\qquad a^0 = 1$

$(ab)^x = a^x \cdot b^x$ $\qquad \left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

3) Identities: $a^{\log_a x} = x$ $(x > 0)$, $\qquad \log_a a^x = x$ (x is any real number)

4) Properties of logarithms ($x > 0$, $y > 0$, $a > 0$, and $a \neq 1$):

(a) $\log_a(xy) = \log_a x + \log_a y$ $\qquad$ (b) $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

(c) $\log_a x^r = r \log_a x$ $\qquad$ (d) $\log_a a = 1$

(e) $\log_a 1 = 0$

5) Change-of-Base Theorem: $\log_a x = \dfrac{\log_b x}{\log_b a}$ $\qquad (a, b, x > 0 \quad a \neq 1 \quad b \neq 1)$

6) Exponential Growth or Decay: $\qquad y = y_0 e^{kt}$

where y_0 is the amount presented at time $t = 0$, y is the amount presented at time t, and k is a constant.

1. Sketch the graph of the parabola $f(x) = -2x^2 + 8x - 8$. Find the vertex, axis, and intercepts. Is it open up or down? Write the domain and range. Sketch the graph.

2. A ball was thrown upward with initial velocity $v_0 = 80$ ft/sec from a bridge of height $s_0 = 96$ ft. above a river.

a) Write the expression for the height $s(t)$ of the ball above the water. [Recall: $s(t) = -16t^2 + v_0 t + s_0$ (in feet)].

b) What is the maximum height of the ball?

c) When does the ball reach this height?

d) After how many seconds will the ball fall into the river?

e) Sketch the graph of the function $s = s(t)$.

3. A furniture outlet has been selling 32 sofas a week at $900 each. The past statistics indicate that for each $50 rebate, the number of sofas sold will increase by 8 per week. The cost function for selling x units is $C(x) = 7500 + 500x$.

(a) Express the price p as a linear function of the demand x.

(b) What price should be charged to sell 44 sofas per week?

(c) Find the profit function $P(x)$.

(d) Find the production level that will maximize profit.

(e) What price will maximize the profit?

4. A farmer uses 1200 feet of fencing to enclose a rectangular region and also subdivide the region into three smaller equal rectangular regions by placing two fences parallel to one of the sides. What is the largest total area that can be enclosed?

5. Find all zeros of $g(x)=\left(x^3-4x\right)\left(x^2-5x+6\right)^2\left(x^2+4\right)^3$ and their multiplicities (Write the multiplicity in parentheses next to the zero.) Which of the zeros are the $x-$intercepts?

6. Make a rough sketch of the graph of the function $y=-2(x-1)(3x+4)^2$. Determine the function's behavior when x becomes infinitely large (positive or negative).

7. Sketch the graph of the rational function $f(x)=\dfrac{(x-2)\left(x^2-4x+3\right)}{\left(x^2-6x+8\right)(x-1)^2}$.

(State the domain. Find all holes, asymptotes, intercepts and points where the graph crosses its horizontal asymptote. If any of the above does not exist, write: None).

8. Which of the following functions $f, g,$ and h are 1-1: $f(x)=\sqrt{x+5}$, $g(x)=x^2-x$, $h(x)=\dfrac{1}{x-1}$?

9. If $g(x)=2x-6$, $f(4)=5$, and f is 1-1, find $\left(g\circ f^{-1}\right)(5)$.

10. Without finding f^{-1}, sketch its graph if $f(x)=\sqrt{x}\ -1$.

11. Find $f^{-1}(x)$ and its domain and range if:

a) $f(x)=\dfrac{x-5}{x+2}$ b) $f(x)=-3\sqrt{x-4}+5$ c) $f(x)=x^2-2x,\quad x\geq 1$

12. Sketch the graph of the function $f(x)=2-e^{-x+1}$. Find its domain, range and asymptote. Is function increasing or decreasing?

13. Solve the equations:

a) $\dfrac{4^x}{2}=\left(\dfrac{1}{2}\right)^{x^2-7}$ b) $3\cdot 9^{x-2}+2\cdot 3^{x-2}-1=0$.

14. George wants to buy a \$30,000 car. He has saved \$27,000. What annual interest rate would be required for his \$27,000 to grow to \$30,000 in 3 years?

(Recall: $A=P\left(1+\dfrac{r}{m}\right)^{mt}$)

15. Find the domain, range, asymptote, intercepts and sketch the graph of the function $y=2-2\ln(x+1)$.

16. Write the statements below in equivalent exponential form:

a) $\log_5 125 = 3$ b) $\ln 2 = x$ c) $\log \frac{1}{1000} = -3$.

17. Write the statements below in equivalent logarithmic form:

a) $e^0 = 1$ b) $10^{2x} = 4$ c) $\left(\frac{1}{2}\right)^{-4} = 16$ d) $\frac{1}{y} = 2^{1-x}$

18. Evaluate:

a) $\log_{25} 5$ b) $\log_2 \frac{1}{2}$ c) $\log 100$ d) $\ln e^3$ e) $\log_{\frac{1}{3}} 81$

19. Simplify, where it is possible. Give all restrictions on the variables:

a) $\frac{\log_3 9}{\log_5 25}$ b) $e^{3\ln x}$ c) $\log 10^{2x^2}$ d) $\log_4 2^{x^2}$ e) $5^{\log_3 x}$ f) $\ln\left(x^2 + y^2\right)$

20. Write as a logarithm of a single quantity: $5\log 2 - 3\log x + 2\log y$.

21. Write the single logarithm as a sum and/or difference: $\log_6 \frac{6\sqrt{x}\ y\sqrt{y}}{z^2}$.

22. Use Change-of-Base Theorem to rewrite each logarithm with a new base b. Simplify, where it is possible.

a) $\log_2 e$; $b = e$ b) $\log_{100} x$; $b = 10$ c) $\log_{36} 216$; $b = 6$

23. Given $f(x) = \log_5(2x+1)$, evaluate $f(2)$.

24. Solve the exponential equations:

a) $3\left(2^{x^2}\right) = 99$ b) $e^{3x} = 125$ c) $5^{x-1} = 4^{x+1}$ (use natural logarithm) d) $e^{\frac{x}{3}\ln 4} = 6$

25. Solve the logarithmic equations:

a) $\log_2 (x-3)^2 = 4$ b) $\log_3 (2x+1) - \log_3 (x-1) = 2$

c) $\ln x + \ln(x-2) = \ln 3$ d) $\log_3 \sqrt{x-6} = 2$

26. Find the domain of the function $f(x) = \log\left(x^2 - 9\right)$.

27. Find the inverse function $f^{-1}(x)$ for $f(x)$ given below. Write the domain and range.

a) $f(x) = 4 - e^{2-x}$ b) $f(x) = \log_3(x-2) + 4$

28. Which of the following is/are true?

(a) $e^{\frac{1}{2}\ln x} = \sqrt{x},\ x > 0$ (b) $\log_3 3^{x^2} = x^2$ (c) $\log_5\left(\frac{1}{3}\right) = -\log_5 3$

(d) $\log_2(2x+y) = \log_2 2x + \log_2 y,\quad x > 0, y > 0$ (e) $\ln\left(\frac{5x^2+2}{x^2+1}\right) = \frac{\ln\left(5x^2+2\right)}{\ln(x^2+1)}$

29. a) Find the doubling time of a certain bacteria population if it grows according to the function $y = y_0 e^{3t(\ln 2)/2}$, where t is in hours.

b) Find the half-life of a certain element if it decays according to the function $y = y_0 e^{-t(\ln 2)/200}$, where t is in years.

30. A certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria. Assume that population grows exponentially.
a) Find the formula that gives population of bacteria in t hours.
b) Predict the size of the population in 24 hours.

31. The population of the United States in 1980 was 227 million; in 1990 it was 250 million. Assume that the population grows exponentially.
a) Find the formula that gives population in t years after 1980.
b) Estimate the size of the population in 2004.
c) When will the population reach 300 million?