

L3

§1.4 Factoring

Factoring – the process of finding polynomials whose product is equal to a given polynomial.

A polynomial is considered to be *factored completely* if it is written as a product of *prime* or *irreducible* polynomials.

Factoring out the GCF (*Greatest Common Factor*):

Use the distributive property to factor out the GCF. The GCF is the product of the largest common factor of all numbers and all common factors containing variables, each with the smallest exponent that appears on that factor.

Example. Factor out the GCF from each polynomial:

a) $8x^5y^3 + 6xy^9$

b) $5x^2(x-2)^3 + x(x-2)^2$

Factoring out a -1 :

Example. Use the distributive property to simplify:

$$-(2x^2 - 3x + 4) =$$

By using the distributive property, we can factor a -1 out of a polynomial by changing the sign of each term.

Example. Factor a -1 out of each polynomial:

a) $x - 2$

b) $-3x^2 + 4x - 5$

Factoring by grouping:

Sometimes this method works when we have a polynomial with more than 3 terms. To apply, collect the terms into two or more groups so that each group has a common factor.

Example.

a) $ax^2 + 7a + 3x^2 + 21$

b) $3p^4 - 2np^4 + 2nq - 3q$

Trial and Error Method:

Trial and Error Method is FOIL “in reverse”.

Example. Factor each trinomial:

a) $x^2 - 6x + 5$

b) $14x^2 + 11xy - 15y^2$

c) $6x^2 - 7x - 20$

Note: See Supplemental 2 for more details and method of factoring by using the *Quadratic Formula*.

Perfect square trinomials:

Try to use this method when the first and last terms are perfect squares. Check the middle term and apply one of the formulas below if it is the case:

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Example.

a) $4x^2 + 28x + 49$

b) $9m^2n^2 - 12mn + 4$

Difference of two squares: $x^2 - y^2 = (x - y)(x + y)$

Example.

a) $49x^2 - 81$

b) $16x^4 - 1$

Caution! $x^2 + y^2$ cannot be factored.

Difference and sum of two cubes:

(New formulas to memorize!)

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Example. Factor each polynomial:

a) $8c^3 + 27$

b) $(x - 1)^3 - 64$

Example. Factor by the *Method of Substitution*.

a) $16(x + 1)^2 + 8(x + 1) + 1$

b) $x^6 + 7x^3 - 8$

Example. Factor completely by any method. Factor the GCF first when it is appropriate.

a) $b^6 - 27$

b) $2x^3 - 98x$

c) $m^2 + 6m + 9 - n^2$

d) $4x^2 - 35x + 49$

e) $12x^2(3x-1)^3 + 2x(3x-1)^4$